

Ghost-free $F(R)$ bigravity and accelerating cosmology

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We propose a bigravity analogue of the $F(R)$ gravity. Our construction is based on recent ghost-free massive bigravity where additional scalar fields are added and the corresponding conformal transformation is implemented. It turns out that $F(R)$ bigravity is easier to formulate in terms of the auxiliary scalars as the explicit presentation in terms of $F(R)$ is quite cumbersome. The consistent cosmological reconstruction scheme of $F(R)$ bigravity is developed in detail, showing the possibility to realize nearly arbitrary physical universe evolution with consistent solution for second metric. The examples of accelerating universe which includes phantom, quintessence and Λ CDM acceleration are worked out in detail and their physical properties are briefly discussed.

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I. INTRODUCTION

The formulation of massive spin-two field or massive graviton has a long history initiated from the free field formulation by Fierz and Pauli [1] (for recent review, see [2]). In spite of the success of the free theory, it has been known that there appears the Boulware-Deser ghost [3] in the naive non-linear extension of the Fierz-Pauli formulation. Furthermore, it has been also known that there appears a discontinuity in the limit of $m \rightarrow 0$ in the free massive gravity compared with the Einstein gravity. This discontinuity is due to the extra degrees of freedom in the limit and is called vDVZ (van Dam, Veltman, and Zakharov) discontinuity [4]. The extra degrees of freedom can be screened by the non-linearity, which becomes strong when m is small. Such mechanism is called the Vainshtein mechanism [5]. A similar mechanism works [6] for the bending mode of the so-called DGP model [7]. Moreover, the scalar field models, where the Vainshtein mechanism works, have been proposed.

Recently, there has been much progress in the non-linear formulation of the massive gravity [8, 9] without the Boulware-Deser ghost [3]. Although the corresponding formulation of massive spin-two field is given in the fixed or non-dynamical background metric, the ghost-free model with the dynamical metric has been also proposed [10] (for the recent cosmological aspects of massive ghost-free and bigravity models, see [9, 11]). Since the corresponding model contains two kinds of symmetric tensor fields, the model is called bi-metric gravity or bigravity. The massive gravity was applied in Ref. [12] to explain the current accelerating expansion of the universe. The accelerating cosmology in terms of the recent formulation of the ghost-free bigravity was discussed in [13].

It is commonly accepted nowadays that the expansion of the current universe is accelerating. This was confirmed by the observation of the type Ia supernovae at the end of the last century [14]. In order that the current cosmic acceleration could occur in the Einstein gravity, we need the mysterious cosmological fluid with the negative pressure called dark energy (for recent review, see [15]). The simplest Λ CDM model of dark energy is composed of the cosmological term and CDM (cold dark matter) in the Einstein gravity. The Λ CDM model, however, suffers from the so-called fine-tuning problem and/or coincidence problem. In order to avoid these problems, many kinds of dynamical models have been proposed.

Among such dynamical models, much attention has been given to the so-called $F(R)$ gravity which was proposed as gravitational alternative for cosmic acceleration in Refs. [17, 18] (for recent review, see [16]). In $F(R)$ gravity, the scalar curvature R in the Einstein-Hilbert action is replaced by an appropriate function $F(R)$ of the scalar curvature. In this paper, we propose a bigravity analogue of the $F(R)$ gravity. We formulate the theory which respects the desirable properties of the recent bigravity models and for example, the Boulware-Deser ghost does not appear. It is demonstrated that the obtained field equations are consistent with each other and consistent cosmological solutions

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can be obtained. Furthermore, we show that a wide class of the cosmological solutions, including the accelerated expanding universe, can be realized in this formulation. Therefore, the models under consideration have much richer structure than simple bigravity recently investigated in [13].

II. GHOST-FREE $F(R)$ BIGRAVITY

A model of bi-metric gravity, which includes two metric tensors $g_{\mu\nu}$ and $f_{\mu\nu}$, was proposed in Ref. [10]. The model describes the massless spin-two field, corresponding to graviton, and massive spin-two field. It has been shown that the Boulware-Deser ghost [3] does not appear in such a theory.

The action is given by

$$S_{\text{bi}} = M_g^2 \int d^4x \sqrt{-\det g} R^{(g)} + M_f^2 \int d^4x \sqrt{-\det f} R^{(f)} \\ + 2m^2 M_{\text{eff}}^2 \int d^4x \sqrt{-\det g} \sum_{n=0}^4 \beta_n e_n \left(\sqrt{g^{-1}f} \right). \quad (1)$$

Here $R^{(g)}$ is the scalar curvature for $g_{\mu\nu}$ and $R^{(f)}$ is the scalar curvature for $f_{\mu\nu}$. The tensor $\sqrt{g^{-1}f}$ is defined by the square root of $g^{\mu\rho} f_{\rho\nu}$, that is, $\left(\sqrt{g^{-1}f}\right)_{\rho}^{\mu} \left(\sqrt{g^{-1}f}\right)_{\nu}^{\rho} = g^{\mu\rho} f_{\rho\nu}$. For the tensor X_{ν}^{μ} , $e_n(X)$'s are defined by

$$e_0(X) = 1, \quad e_1(X) = [X], \quad e_2(X) = \frac{1}{2}([X]^2 - [X^2]), \\ e_3(X) = \frac{1}{6}([X]^3 - 3[X][X^2] + 2[X^3]), \\ e_4(X) = \frac{1}{24}([X]^4 - 6[X]^2[X^2] + 3[X^2]^2 + 8[X][X^3] - 6[X^4]), \\ e_k(X) = 0 \quad \text{for } k > 4. \quad (2)$$

Here $[X]$ expresses the trace of X : $[X] = X_{\mu}^{\mu}$.

We now construct a bigravity model analogous to the $F(R)$ gravity. Before going to the explicit construction, one may review the scalar-tensor description of the usual $F(R)$ gravity [18]. In $F(R)$ gravity, the scalar curvature R in the Einstein-Hilbert action

$$S_{\text{EH}} = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa^2} + \mathcal{L}_{\text{matter}} \right), \quad (3)$$

is replaced by an appropriate function of the scalar curvature:

$$S_{F(R)} = \int d^4x \sqrt{-g} \left(\frac{F(R)}{2\kappa^2} + \mathcal{L}_{\text{matter}} \right). \quad (4)$$

One can also rewrite $F(R)$ gravity in the scalar-tensor form. By introducing the auxiliary field A , the action (4) of the $F(R)$ gravity is rewritten in the following form:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \{ F'(A) (R - A) + F(A) \}. \quad (5)$$

By the variation of A , one obtains $A = R$. Substituting $A = R$ into the action (5), one can reproduce the action in (4). Furthermore, we rescale the metric in the following way (conformal transformation):

$$g_{\mu\nu} \rightarrow e^{\sigma} g_{\mu\nu}, \quad \sigma = -\ln F'(A). \quad (6)$$

Thus, the Einstein frame action is obtained:

$$S_E = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(R - \frac{3}{2} g^{\rho\sigma} \partial_{\rho} \sigma \partial_{\sigma} \sigma - V(\sigma) \right), \\ V(\sigma) = e^{\sigma} g(e^{-\sigma}) - e^{2\sigma} f(g(e^{-\sigma})) = \frac{A}{F'(A)} - \frac{F(A)}{F'(A)^2}. \quad (7)$$

Here $g(e^{-\sigma})$ is given by solving the equation $\sigma = -\ln(1 + f'(A)) = -\ln F'(A)$ as $A = g(e^{-\sigma})$. Due to the scale transformation (6), the scalar field σ couples usual matter.

In order to construct a model analogous to the $F(R)$ gravity, we added the following action to the action (1):

$$S_1 = -M_g^2 \int d^4x \sqrt{-\det g} \left\{ \frac{3}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + V(\varphi) \right\} + \int d^4x \mathcal{L}_{\text{matter}}(e^\varphi g_{\mu\nu}, \Phi_i). \quad (8)$$

Here we denote the matter field by Φ_i . As discussed in [10], the action (8) does not break the good properties like the absence of the Boulware-Deser ghost.

By the conformal transformation $g_{\mu\nu} \rightarrow e^{-\varphi} g_{\mu\nu}$, the total action $S_{\text{total}} = S_{\text{bi}} + S_1$ is transformed to

$$\begin{aligned} S_{\text{total}} &\rightarrow \\ S_{FR} &= M_f^2 \int d^4x \sqrt{-\det f} R^{(f)} + 2m^2 M_{\text{eff}}^2 \int d^4x \sqrt{-\det g} \sum_{n=0}^4 \beta_n e^{(\frac{n}{2}-2)\varphi} e_n(\sqrt{g^{-1}f}) \\ &\quad + M_g^2 \int d^4x \sqrt{-\det g} \left\{ e^{-\varphi} R^{(g)} + e^{-2\varphi} V(\varphi) \right\} + \int d^4x \mathcal{L}_{\text{matter}}(g_{\mu\nu}, \Phi_i). \end{aligned} \quad (9)$$

Then the kinetic term of φ and the coupling of φ with matter disappear. By the variation over φ , we obtain

$$0 = 2m^2 M_{\text{eff}}^2 \sum_{n=0}^4 \beta_n \left(\frac{n}{2} - 2 \right) e^{(\frac{n}{2}-2)\varphi} e_n(\sqrt{g^{-1}f}) + M_g^2 \left\{ -e^{-\varphi} R^{(g)} - 2e^{-2\varphi} V(\varphi) + e^{-2\varphi} V'(\varphi) \right\}. \quad (10)$$

Eq. (10) can be solved algebraically with respect to φ as $\varphi = \varphi(R^{(g)}, e_n(\sqrt{g^{-1}f}))$. Then by substituting the expression of φ into (9), a model analogous to the $F(R)$ gravity follows:

$$\begin{aligned} S_{FR} &= M_f^2 \int d^4x \sqrt{-\det f} R^{(f)} \\ &\quad + 2m^2 M_{\text{eff}}^2 \int d^4x \sqrt{-\det g} \sum_{n=0}^4 \beta_n e^{(\frac{n}{2}-2)\varphi(R^{(g)}, e_n(\sqrt{g^{-1}f}))} e_n(\sqrt{g^{-1}f}) \\ &\quad + M_g^2 \int d^4x \sqrt{-\det g} F(R^{(g)}, e_n(\sqrt{g^{-1}f})) + \int d^4x \mathcal{L}_{\text{matter}}(g_{\mu\nu}, \Phi_i), \\ F(R^{(g)}, e_n(\sqrt{g^{-1}f})) &\equiv e^{-\varphi(R^{(g)}, e_n(\sqrt{g^{-1}f}))} R^{(g)} + e^{-2\varphi(R^{(g)}, e_n(\sqrt{g^{-1}f}))} V(\varphi(R^{(g)}, e_n(\sqrt{g^{-1}f}))). \end{aligned} \quad (11)$$

Note that it is difficult to solve (10) with respect to φ explicitly. Therefore, it might be better to define the model by introducing the auxiliary scalar field φ as in (9). Of course, in some cases $F(R^{(g)}, e_n(\sqrt{g^{-1}f}))$ can be explicitly found. For instance, in the minimal case, where $\beta_0 = 3$, $\beta_1 = -1$, $\beta_2 = \beta_3 = 0$, and $\beta_4 = 1$, one may consider the simplest case $V = V_0 e^{-\varphi}$ with a constant V_0 . Then Eq. (10) reduces to

$$0 = m^2 M_{\text{eff}}^2 \left(-12e^{-2\varphi} e_0(\sqrt{g^{-1}f}) + 3e^{-\frac{3}{2}\varphi} e_1(\sqrt{g^{-1}f}) \right) - M_g^2 e^{-\varphi} (R^{(g)} + V_0), \quad (12)$$

which can be solved with respect to $e^{-\frac{\varphi}{2}}$ as

$$e^{-\frac{\varphi}{2}} = \frac{e_1(\sqrt{g^{-1}f})}{8e_0(\sqrt{g^{-1}f})} \pm \sqrt{\frac{e_1(\sqrt{g^{-1}f})^2}{64e_0(\sqrt{g^{-1}f})^2} - \frac{M_g^2 e^{-\varphi} (R^{(g)} + V_0)}{12m^2 M_{\text{eff}}^2 e^{-2\varphi} e_0(\sqrt{g^{-1}f})}}, \quad (13)$$

and we obtain

$$\begin{aligned} F(R^{(g)}, e_n(\sqrt{g^{-1}f})) &\equiv \left(\frac{e_1(\sqrt{g^{-1}f})}{8e_0(\sqrt{g^{-1}f})} \pm \sqrt{\frac{e_1(\sqrt{g^{-1}f})^2}{64e_0(\sqrt{g^{-1}f})^2} - \frac{M_g^2 e^{-\varphi} (R^{(g)} + V_0)}{12m^2 M_{\text{eff}}^2 e^{-2\varphi} e_0(\sqrt{g^{-1}f})}} \right)^2 R^{(g)} \\ &\quad + \left(\frac{e_1(\sqrt{g^{-1}f})}{8e_0(\sqrt{g^{-1}f})} \pm \sqrt{\frac{e_1(\sqrt{g^{-1}f})^2}{64e_0(\sqrt{g^{-1}f})^2} - \frac{M_g^2 e^{-\varphi} (R^{(g)} + V_0)}{12m^2 M_{\text{eff}}^2 e^{-2\varphi} e_0(\sqrt{g^{-1}f})}} \right)^4 \end{aligned}$$

$$\times V \left(-2 \ln \left(\frac{e_1 \left(\sqrt{g^{-1}f} \right)}{8e_0 \left(\sqrt{g^{-1}f} \right)} \pm \sqrt{\frac{e_1 \left(\sqrt{g^{-1}f} \right)^2}{64e_0 \left(\sqrt{g^{-1}f} \right)^2} - \frac{M_g^2 e^{-\varphi} \left(R^{(g)} + V_0 \right)}{12m^2 M_{\text{eff}}^2 e^{-2\varphi} e_0 \left(\sqrt{g^{-1}f} \right)}} \right) \right) \quad (14)$$

Hence, we may define the analogue of the $F(R)$ gravity by (9).

Even for the sector including $f_{\mu\nu}$, one may consider the analogue of the $F(R)$ gravity by adding the action of another scalar field ξ as follows:

$$S_\xi = -M_f^2 \int d^4x \sqrt{-\det f} \left\{ \frac{3}{2} f^{\mu\nu} \partial_\mu \xi \partial_\nu \xi + U(\xi) \right\}. \quad (15)$$

By the conformal transformation for $f_{\mu\nu}$: $f_{\mu\nu} \rightarrow e^{-\xi} f_{\mu\nu}$, instead of (9), we obtain

$$\begin{aligned} S_F = & M_f^2 \int d^4x \sqrt{-\det f} \left\{ e^{-\xi} R^{(f)} + e^{-2\xi} U(\xi) \right\} \\ & + 2m^2 M_{\text{eff}}^2 \int d^4x \sqrt{-\det g} \sum_{n=0}^4 \beta_n e^{\left(\frac{n}{2}-2\right)\varphi - \frac{n}{2}\xi} e_n \left(\sqrt{g^{-1}f} \right) \\ & + M_g^2 \int d^4x \sqrt{-\det g} \left\{ e^{-\varphi} R^{(g)} + e^{-2\varphi} V(\varphi) \right\} + \int d^4x \mathcal{L}_{\text{matter}}(g_{\mu\nu}, \Phi_i). \end{aligned} \quad (16)$$

Again the kinetic term of ξ vanishes and by the variation of φ and ξ , we obtain

$$0 = 2m^2 M_{\text{eff}}^2 \sum_{n=0}^4 \beta_n \left(\frac{n}{2} - 2 \right) e^{\left(\frac{n}{2}-2\right)\varphi - \frac{n}{2}\xi} e_n \left(\sqrt{g^{-1}f} \right) + M_g^2 \left\{ -e^{-\varphi} R^{(g)} - 2e^{-2\varphi} V(\varphi) + e^{-2\varphi} V'(\varphi) \right\}, \quad (17)$$

$$0 = -2m^2 M_{\text{eff}}^2 \sum_{n=0}^4 \frac{\beta_n n}{2} e^{\left(\frac{n}{2}-2\right)\varphi - \frac{n}{2}\xi} e_n \left(\sqrt{g^{-1}f} \right) + M_f^2 \left\{ -e^{-\xi} R^{(f)} - 2e^{-2\xi} U(\xi) + e^{-2\xi} U'(\xi) \right\}. \quad (18)$$

The obtained equations (17) and (18) can be solved algebraically with respect to φ and ξ as $\varphi = \varphi \left(R^{(g)}, R^{(f)}, e_n \left(\sqrt{g^{-1}f} \right) \right)$ and $\xi = \xi \left(R^{(g)}, R^{(f)}, e_n \left(\sqrt{g^{-1}f} \right) \right)$. Substituting the expression of φ and ξ into (16), we obtain a model analogous to the $F(R)$ gravity:

$$\begin{aligned} S_F = & M_f^2 \int d^4x \sqrt{-\det f} F^{(f)} \left(R^{(g)}, R^{(f)}, e_n \left(\sqrt{g^{-1}f} \right) \right) \\ & + 2m^2 M_{\text{eff}}^2 \int d^4x \sqrt{-\det g} \sum_{n=0}^4 \beta_n e^{\left(\frac{n}{2}-2\right)\varphi \left(R^{(g)}, e_n \left(\sqrt{g^{-1}f} \right) \right)} e_n \left(\sqrt{g^{-1}f} \right) \\ & + M_g^2 \int d^4x \sqrt{-\det g} F^{(g)} \left(R^{(g)}, R^{(f)}, e_n \left(\sqrt{g^{-1}f} \right) \right) + \int d^4x \mathcal{L}_{\text{matter}}(g_{\mu\nu}, \Phi_i), \\ F^{(g)} \left(R^{(g)}, R^{(f)}, e_n \left(\sqrt{g^{-1}f} \right) \right) \equiv & \left\{ e^{-\varphi \left(R^{(g)}, R^{(f)}, e_n \left(\sqrt{g^{-1}f} \right) \right)} R^{(g)} \right. \\ & \left. + e^{-2\varphi \left(R^{(g)}, R^{(f)}, e_n \left(\sqrt{g^{-1}f} \right) \right)} V \left(\varphi \left(R^{(g)}, R^{(f)}, e_n \left(\sqrt{g^{-1}f} \right) \right) \right) \right\}, \\ F^{(f)} \left(R^{(g)}, R^{(f)}, e_n \left(\sqrt{g^{-1}f} \right) \right) \equiv & \left\{ e^{-\xi \left(R^{(g)}, R^{(f)}, e_n \left(\sqrt{g^{-1}f} \right) \right)} R^{(f)} \right. \\ & \left. + e^{-2\xi \left(R^{(g)}, R^{(f)}, e_n \left(\sqrt{g^{-1}f} \right) \right)} U \left(\xi \left(R^{(g)}, R^{(f)}, e_n \left(\sqrt{g^{-1}f} \right) \right) \right) \right\}. \end{aligned} \quad (19)$$

We should again note that it is difficult to explicitly solve (17) and (18) with respect to φ and ξ and it might be better to define the model by introducing the auxiliary scalar fields φ and ξ as in (16).

Hence, we succeeded to obtain the bigravity analogue of the $F(R)$ gravity.

III. COSMOLOGICAL RECONSTRUCTION

We now consider the minimal case, where

$$S_{\text{bi}} = M_g^2 \int d^4x \sqrt{-\det g} R^{(g)} + M_f^2 \int d^4x \sqrt{-\det f} R^{(f)} \\ + 2m^2 M_{\text{eff}}^2 \int d^4x \sqrt{-\det g} \left(3 - \text{tr} \sqrt{g^{-1}f} + \det \sqrt{g^{-1}f} \right). \quad (20)$$

In order to evaluate $\delta\sqrt{g^{-1}f}$, we consider two matrices M and N , which satisfy the relation $M^2 = N$. Since $\delta M M + M \delta M = \delta N$, we find

$$\delta M = \delta N M^{-1} - M \delta M M^{-1}. \quad (21)$$

By using (21) iteratively, one obtains

$$\delta M = \delta N M^{-1} - M \delta M M^{-1} = \delta N M^{-1} - M \delta N M^{-2} + M^2 \delta M M^{-2} = \sum_{n=0} (-1)^n M^n \delta N M^{-n-1}. \quad (22)$$

Then by carefully considering the trace of Eq. (21), we find

$$\text{tr} \delta M = \frac{1}{2} \text{tr} (M^{-1} \delta N). \quad (23)$$

For a while, we work in the Einstein frame action (20) with (8) and (15) but the contribution from the matter is neglected. Then by the variation of $g_{\mu\nu}$, one obtains

$$0 = M_g^2 \left(\frac{1}{2} g_{\mu\nu} R^{(g)} - R_{\mu\nu}^{(g)} \right) + m^2 M_{\text{eff}} \left\{ g_{\mu\nu} \left(3 - \text{tr} \sqrt{g^{-1}f} \right) + f_{\mu\rho} \left(\sqrt{g^{-1}f} \right)^{-1\rho}_{\nu} \right\} \\ + \frac{1}{2} \left(\frac{3}{2} g^{\rho\sigma} \partial_\rho \varphi \partial_\sigma \varphi + V(\varphi) \right) g_{\mu\nu} - \frac{3}{2} \partial_\mu \varphi \partial_\nu \varphi. \quad (24)$$

On the variation of $f_{\mu\nu}$, we obtain

$$0 = M_f^2 \left(\frac{1}{2} f_{\mu\nu} R^{(f)} - R_{\mu\nu}^{(f)} \right) + m^2 M_{\text{eff}} \left\{ f_{\mu\nu} \left(3 - \text{tr} \sqrt{g^{-1}f} \right) - f_{\mu\sigma} \left(\sqrt{g^{-1}f} \right)^{-1\sigma}_{\rho} g^{\rho\tau} f_{\tau\nu} \right\} \\ + \frac{1}{2} \left(\frac{3}{2} f^{\rho\sigma} \partial_\rho \xi \partial_\sigma \xi + U(\xi) \right) f_{\mu\nu} - \frac{3}{2} \partial_\mu \xi \partial_\nu \xi. \quad (25)$$

We now assume the FRW universes for the metrics $g_{\mu\nu}$ and $f_{\mu\nu}$:

$$ds_g^2 = \sum_{\mu,\nu=0}^3 g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a(t)^2 \sum_{i=1}^3 (dx^i)^2, \quad ds_f^2 = \sum_{\mu,\nu=0}^3 f_{\mu\nu} dx^\mu dx^\nu = -dt^2 + b(t)^2 \sum_{i=1}^3 (dx^i)^2. \quad (26)$$

Then the (t, t) component of (24) gives

$$0 = -3M_g^2 H^2 - 3m^2 M_{\text{eff}}^2 \left(1 - \frac{b}{a} \right) - \frac{3}{4} \dot{\varphi}^2 - \frac{1}{2} V(\varphi), \quad (27)$$

and (i, j) components give

$$0 = M_g^2 \left(2\dot{H} + 3H^2 \right) + 2m^2 M_{\text{eff}}^2 \left(1 - \frac{b}{a} \right) - \frac{3}{4} \dot{\varphi}^2 + \frac{1}{2} V(\varphi). \quad (28)$$

Here $H = \dot{a}/a$. On the other hand, the (t, t) component of (25) gives

$$0 = -3M_f^2 K^2 - m^2 M_{\text{eff}}^2 \left(1 - \frac{3b}{a} \right) - \frac{3}{4} \dot{\xi}^2 - \frac{1}{2} U(\xi), \quad (29)$$

and (i, j) components give

$$0 = M_f^2 \left(2\dot{K} + 3K^2 \right) + 2m^2 M_{\text{eff}}^2 \left(1 - \frac{2b}{a} \right) - \frac{3}{4}\dot{\xi}^2 + \frac{1}{2}U(\xi). \quad (30)$$

Here $K = \dot{b}/b$. Hence,

$$0 = 2M_g^2 \dot{H} - m^2 M_{\text{eff}}^2 \left(1 - \frac{b}{a} \right) - \frac{3}{2}\dot{\varphi}^2, \quad (31)$$

$$0 = -M_g^2 \left(2\dot{H} + 6H^2 \right) - 5m^2 M_{\text{eff}}^2 \left(1 - \frac{b}{a} \right) - V(\varphi), \quad (32)$$

$$0 = 2M_f^2 \dot{K} + m^2 M_{\text{eff}}^2 \left(1 - \frac{b}{a} \right) - \frac{3}{2}\dot{\xi}^2, \quad (33)$$

$$0 = -M_f^2 \left(2\dot{K} + 6K^2 \right) - m^2 M_{\text{eff}}^2 \left(3 - \frac{7b}{a} \right) - U(\xi). \quad (34)$$

One now redefines scalar fields as $\varphi = \varphi(\eta)$ and $\xi = \xi(\zeta)$ and identify η and ζ with the cosmological time t . Then we find

$$\frac{\omega(t)}{2} = 2M_g^2 \dot{H} - m^2 M_{\text{eff}}^2 \left(1 - \frac{b}{a} \right), \quad (35)$$

$$\tilde{V}(t) = -M_g^2 \left(2\dot{H} + 6H^2 \right) - 5m^2 M_{\text{eff}}^2 \left(1 - \frac{b}{a} \right), \quad (36)$$

$$\frac{\sigma(t)}{2} = 2M_f^2 \dot{K} + m^2 M_{\text{eff}}^2 \left(1 - \frac{b}{a} \right), \quad (37)$$

$$\tilde{U}(t) = -M_f^2 \left(2\dot{K} + 6K^2 \right) - m^2 M_{\text{eff}}^2 \left(3 - \frac{7b}{a} \right). \quad (38)$$

Here

$$\omega(\eta) = 3\varphi'(\eta)^2, \quad \tilde{V}(\eta) = V(\varphi(\eta)), \quad \sigma(\zeta) = 3\xi'(\zeta)^2, \quad \tilde{U}(\zeta) = U(\xi(\zeta)). \quad (39)$$

Then for arbitrary $a(t)$ and $b(t)$, if we choose $\omega(t)$, $\tilde{V}(t)$, $\sigma(t)$, and $\tilde{U}(t)$ to satisfy Eqs. (35-38), a model admitting the given $a(t)$ and $b(t)$ evolution can be reconstructed.

Consider the possibility not to introduce the extra scalar field χ (15). Instead of the introduction of χ , we assume the metric $f_{\mu\nu}$ in the following form:

$$ds_f^2 = \sum_{\mu, \nu=0}^3 f_{\mu\nu} dx^\mu dx^\nu = -c(t)^2 dt^2 + b(t)^2 \sum_{i=1}^3 (dx^i)^2. \quad (40)$$

Then instead of Eqs. (27-30), one gets

$$0 = -3M_g^2 H^2 - 3m^2 M_{\text{eff}}^2 \left(1 - \frac{b}{a} \right) - \frac{3}{4}\dot{\varphi}^2 - \frac{1}{2}V(\varphi), \quad (41)$$

$$0 = M_g^2 \left(2\dot{H} + 3H^2 \right) + m^2 M_{\text{eff}}^2 \left(3 - c - 2\frac{b}{a} \right) - \frac{3}{4}\dot{\varphi}^2 + \frac{1}{2}V(\varphi), \quad (42)$$

$$0 = -3M_f^2 K^2 - m^2 M_{\text{eff}}^2 \left(3 - 2c - \frac{3b}{a} \right) c^2, \quad (43)$$

$$0 = M_f^2 \left(2\dot{K} + 3K^2 - 2LK \right) + m^2 M_{\text{eff}}^2 \left(3 - c - \frac{4b}{a} \right) c^2. \quad (44)$$

Here $L = \dot{c}/c$.

For a given $a = a(t)$, Eqs. (43) and (44) could be solved with respect to b and c . On the other hand, as in (35) and (36), Eqs. (41) and (42) can be rewritten as

$$\frac{\omega(t)}{2} = 2M_g^2 \dot{H} - m^2 M_{\text{eff}}^2 \left(c - \frac{b}{a} \right), \quad (45)$$

$$\tilde{V}(t) = -M_g^2 \left(2\dot{H} + 6H^2 \right) - m^2 M_{\text{eff}}^2 \left(6 - c - 5\frac{b}{a} \right). \quad (46)$$

Here $\omega(t)$ and $\tilde{V}(t)$ are defined by (39). Then for arbitrary $a(t)$, if we choose $\omega(t)$ and $\tilde{V}(t)$ to satisfy Eqs. (45) and (46), a model admitting the given $a(t)$ can be reconstructed.

IV. EXAMPLES OF ACCELERATING COSMOLOGICAL SOLUTIONS

Let us consider several examples. As discussed around (9), the physical metric, where the scalar field does not directly coupled with matter, is given by multiplying the scalar field to the metric in the Einstein frame in (8) or (20):

$$g_{\mu\nu}^{\text{phys}} = e^\varphi g_{\mu\nu}. \quad (47)$$

The metric of the FRW universe with flat spatial part is conformally flat and therefore given by

$$ds^2 = \tilde{a}(t)^2 \left(-dt^2 + \sum_{i=1}^3 (dx^i)^2 \right). \quad (48)$$

In case $\tilde{a}(t)^2 = \frac{l^2}{t^2}$, the metric (48) corresponds to the de Sitter universe. On the other hand if $\tilde{a}(t)^2 = \frac{l^{2n}}{t^{2n}}$ with $n \neq 1$, by redefining the time coordinate by

$$d\tilde{t} = \pm \frac{l^n}{t^n} dt, \quad (49)$$

that is,

$$\tilde{t} = \pm \frac{l^n}{n-1} t^{1-n}, \quad (50)$$

the metric (48) can be rewritten as

$$ds^2 = -d\tilde{t}^2 + \left(\pm(n-1)\frac{\tilde{t}}{l} \right)^{-\frac{2n}{1-n}} \sum_{i=1}^3 (dx^i)^2. \quad (51)$$

Then if $0 < n < 1$, the metric corresponds to the phantom universe, if $n > 1$ to the quintessence universe, and if $n < 0$ to decelerating universe (For similar scenario in usual non-linear massive gravity, see also [19]). In case of the phantom universe ($0 < n < 1$), one should choose + sign in \pm of (49) or (50) and shift t as $t \rightarrow t - t_0$. Then $t = t_0$ corresponds to the Big Rip and the present time is $t < t_0$ and $t \rightarrow \infty$ to the infinite past ($\tilde{t} \rightarrow -\infty$). In case of the quintessence universe ($n > 1$), we may again choose + sign in \pm of (49) or (50). Then $t \rightarrow 0$ corresponds to $\tilde{t} \rightarrow +\infty$ and $t \rightarrow +\infty$ to $\tilde{t} \rightarrow 0$, which may correspond to the Big Bang. In case of the decelerating universe ($n < 0$), we may choose - sign in \pm of (49) or (50). Then $t \rightarrow 0$ corresponds to $\tilde{t} \rightarrow +\infty$ and $t \rightarrow +\infty$ to $\tilde{t} \rightarrow 0$, which may correspond to the Big Bang, again. We should also note that in case of de Sitter universe ($n = 1$), $t \rightarrow 0$ corresponds to $\tilde{t} \rightarrow +\infty$ and $t \rightarrow \pm\infty$ to $\tilde{t} \rightarrow -\infty$. Let us now choose the metric in the Einstein frame to be flat, where $H = 0$, and

$$e^\varphi = \frac{l^{2n}}{t^{2n}}. \quad (52)$$

Using (39), we find

$$\omega(t) = \frac{12n^2}{t^2}, \quad (53)$$

and Eq. (35) gives

$$b - 1 = \frac{6n^2}{m^2 M_{\text{eff}}^2 t^2}. \quad (54)$$

Eq. (54) shows the behavior of the metric $f_{\mu\nu}$:

$$f_{00} = 1, \quad f_{ij} = b^2 \delta_{ij} = \left(1 + \frac{6n^2}{m^2 M_{\text{eff}}^2 t^2} \right)^2 \delta_{ij}. \quad (55)$$

Then for large t , we find $f_{ij} \rightarrow \delta_{ij}$, that is, the flat metric. On the other hand, for small t

$$f_{ij} \sim \frac{36n^4}{m^4 M_{\text{eff}}^4 t^4}, \quad (56)$$

which becomes larger and larger. Since small t corresponds to large physical time \tilde{t} for the phantom, the de Sitter, and the quintessence universes, the late-time acceleration could be generated by the evolution of $f_{\mu\nu}$.

Using (36), the potential is

$$\tilde{V}(t) = \frac{30n^2}{t^2}. \quad (57)$$

We also find

$$K = \frac{-\frac{12n^2}{m^2 M_{\text{eff}}^2 t^3}}{1 + \frac{6n^2}{m^2 M_{\text{eff}}^2 t^2}}, \quad \dot{K} = \frac{\frac{36n^2}{m^2 M_{\text{eff}}^2 t^4} \left(1 + \frac{2n^2}{m^2 M_{\text{eff}}^2 t^2}\right)}{\left(1 + \frac{6n^2}{m^2 M_{\text{eff}}^2 t^2}\right)^2}. \quad (58)$$

With the help of (37) and (38), we obtain

$$\sigma(t) = \frac{\frac{144M_f^2 n^2}{m^2 M_{\text{eff}}^2 t^4} \left(1 + \frac{2n^2}{m^2 M_{\text{eff}}^2 t^2}\right)}{\left(1 + \frac{6n^2}{m^2 M_{\text{eff}}^2 t^2}\right)^2} - \frac{12n^2}{t^2}, \quad (59)$$

$$\tilde{U}(t) = -\frac{\frac{72n^2 M_f^2}{m^2 M_{\text{eff}}^2 t^4} \left(1 + \frac{14n^2}{m^2 M_{\text{eff}}^2 t^2}\right)}{\left(1 + \frac{6n^2}{m^2 M_{\text{eff}}^2 t^2}\right)^2} + m^2 M_{\text{eff}}^2 \left(4 + \frac{42n^2}{m^2 M_{\text{eff}}^2 t^2}\right). \quad (60)$$

When t is small, $\sigma(t)$ behaves as

$$\sigma(t) \sim \left(\frac{8M_f^2}{m^2 M_{\text{eff}}^2} - 12n^2\right) \frac{1}{t^2}. \quad (61)$$

In order to avoid the ghost, we require $\sigma(t) > 0$, which gives a constraint for the parameters as follows:

$$\frac{2M_f^2}{m^2 M_{\text{eff}}^2} > 3n^2. \quad (62)$$

On the other hand, when t is large, the second term dominates in Eq. (59),

$$\sigma(t) \sim -\frac{12n^2}{t^2}. \quad (63)$$

Therefore, $\sigma(t)$ becomes negative although there does not appear the Boulware-Deser ghost [3], there could appear an additional ghost associated with the scalar field ξ . We should also note the negative σ conflicts with (39) and therefore the model cannot be identified with the analogue of the $F(R)$ gravity. This problem can be, however, avoided by modifying the large t behavior. Indeed, large t does not always mean the late-time when we choose the physical time \tilde{t} in (50) as discussed after Eq. (51). In case of the phantom universe ($0 < n < 1$), $t \rightarrow \infty$ corresponds to the infinite past ($\tilde{t} \rightarrow -\infty$). In case of the quintessence universe ($n > 1$) or the decelerating universe ($n < 0$), the limit of $t \rightarrow +\infty$ corresponds to that of $\tilde{t} \rightarrow 0$. Even in case of de Sitter universe ($n = 1$), $t \rightarrow \pm\infty$ corresponds to $\tilde{t} \rightarrow -\infty$. Therefore, the modification of large t does not affect the late-time behavior of the universe.

Finally, the Λ CDM-like universe may be reconstructed:

$$ds^2 = -d\tilde{t}^2 + A^2 \sinh^3 \frac{\tilde{t}}{l} \sum_{i=1}^3 (dx^i)^2. \quad (64)$$

Here A and l are constants. Changing the time variable \tilde{t} by

$$dt = \frac{d\tilde{t}}{A \sinh^{\frac{3}{2}} \frac{\tilde{t}}{l}}, \quad (65)$$

we obtain the conformal form of the metric as in (48). Eq. (65) gives

$$t = -\frac{l\sqrt{2}}{A}B\left(e^{-\frac{2\tilde{t}}{l}}; \frac{3}{4}, -\frac{1}{2}\right). \quad (66)$$

Here $B(x, a, b)$ is the incomplete beta function defined by

$$B(x, a, b) \equiv \int_0^x dx x^{a-1} (1-x)^{b-1}. \quad (67)$$

Then

$$e^\varphi = \tilde{a}(t)^2 = A^2 \sinh^3 \frac{t(\tilde{t})}{l}, \quad \tilde{t} = -\frac{1}{2} \ln \left(B^{-1} \left(-\frac{At}{l\sqrt{2}} \right); \frac{3}{4}, -\frac{1}{2} \right). \quad (68)$$

Here $B^{-1}(y, a, b)$ is the inverse function of $B(x, a, b)$ defined by $x = B^{-1}(y, a, b)$ for $y = B(x, a, b)$. Eq. (39) gives

$$\omega(t) = \frac{27A^2}{l^2} \sinh \frac{t(\tilde{t})}{l} \cosh^2 \frac{t(\tilde{t})}{l}. \quad (69)$$

Therefore Eqs. (35) and (36) give

$$b = 1 + \frac{27A^2}{2m^2 M_{\text{eff}}^2 l^2} \sinh \frac{t(\tilde{t})}{l} \cosh^2 \frac{t(\tilde{t})}{l}, \quad \tilde{V}(t) = \frac{135A^2}{2l^2} \sinh \frac{t(\tilde{t})}{l} \cosh^2 \frac{t(\tilde{t})}{l} = \frac{135}{2l^2} A^{\frac{4}{3}} e^{\frac{\varphi}{2}} \left(1 - \frac{e^{\frac{2}{3}\varphi}}{A^{\frac{4}{3}}} \right). \quad (70)$$

Here we have used (65) and (68). Hence, we find

$$\begin{aligned} K &= \frac{\frac{27A^3}{2m^2 M_{\text{eff}}^2 l^2} \sinh^{\frac{3}{2}} \frac{t(\tilde{t})}{l} \cosh \frac{t(\tilde{t})}{l} \left(\cosh^2 \frac{t(\tilde{t})}{l} + 2 \sinh \frac{t(\tilde{t})}{l} \right)}{1 + \frac{27A^2}{2m^2 M_{\text{eff}}^2 l^2} \sinh \frac{t(\tilde{t})}{l} \cosh^2 \frac{t(\tilde{t})}{l}}, \\ \dot{K} &= \frac{\frac{27A^4}{2m^2 M_{\text{eff}}^2 l^2} \left(2 \sinh^6 \frac{t(\tilde{t})}{l} + 10 \cosh^2 \frac{t(\tilde{t})}{l} \sinh^4 \frac{t(\tilde{t})}{l} + \frac{3}{2} \cosh^4 \frac{t(\tilde{t})}{l} \sinh^2 \frac{t(\tilde{t})}{l} \right)}{1 + \frac{27A^2}{2m^2 M_{\text{eff}}^2 l^2} \sinh \frac{t(\tilde{t})}{l} \cosh^2 \frac{t(\tilde{t})}{l}} \\ &\quad - \left(\frac{\frac{27A^3}{2m^2 M_{\text{eff}}^2 l^2} \sinh^{\frac{3}{2}} \frac{t(\tilde{t})}{l} \cosh \frac{t(\tilde{t})}{l} \left(\cosh^2 \frac{t(\tilde{t})}{l} + 2 \sinh \frac{t(\tilde{t})}{l} \right)}{1 + \frac{27A^2}{2m^2 M_{\text{eff}}^2 l^2} \sinh \frac{t(\tilde{t})}{l} \cosh^2 \frac{t(\tilde{t})}{l}} \right)^2. \end{aligned} \quad (71)$$

By using (37) and (38), we obtain

$$\begin{aligned} \sigma(t) &= 4M_f^2 \left\{ \frac{\frac{27A^4}{2m^2 M_{\text{eff}}^2 l^2} \left(2 \sinh^6 \frac{t(\tilde{t})}{l} + 10 \cosh^2 \frac{t(\tilde{t})}{l} \sinh^4 \frac{t(\tilde{t})}{l} + \frac{3}{2} \cosh^4 \frac{t(\tilde{t})}{l} \sinh^2 \frac{t(\tilde{t})}{l} \right)}{1 + \frac{27A^2}{2m^2 M_{\text{eff}}^2 l^2} \sinh \frac{t(\tilde{t})}{l} \cosh^2 \frac{t(\tilde{t})}{l}} \right. \\ &\quad \left. - \left(\frac{\frac{27A^3}{2m^2 M_{\text{eff}}^2 l^2} \sinh^{\frac{3}{2}} \frac{t(\tilde{t})}{l} \cosh \frac{t(\tilde{t})}{l} \left(\cosh^2 \frac{t(\tilde{t})}{l} + 2 \sinh \frac{t(\tilde{t})}{l} \right)}{1 + \frac{27A^2}{2m^2 M_{\text{eff}}^2 l^2} \sinh \frac{t(\tilde{t})}{l} \cosh^2 \frac{t(\tilde{t})}{l}} \right)^2 \right\} \\ &\quad - \frac{27A^2}{l^2} \sinh \frac{t(\tilde{t})}{l} \cosh^2 \frac{t(\tilde{t})}{l}, \\ \tilde{U}(t) &= -M_f^2 \left\{ \frac{\frac{27A^4}{m^2 M_{\text{eff}}^2 l^2} \left(2 \sinh^6 \frac{t(\tilde{t})}{l} + 10 \cosh^2 \frac{t(\tilde{t})}{l} \sinh^4 \frac{t(\tilde{t})}{l} + \frac{3}{2} \cosh^4 \frac{t(\tilde{t})}{l} \sinh^2 \frac{t(\tilde{t})}{l} \right)}{1 + \frac{27A^2}{2m^2 M_{\text{eff}}^2 l^2} \sinh \frac{t(\tilde{t})}{l} \cosh^2 \frac{t(\tilde{t})}{l}} \right. \end{aligned}$$

$$\begin{aligned}
& +4 \left(\frac{\frac{27A^3}{2m^2 M_{\text{eff}}^2 l^2} \sinh^{\frac{3}{2}} \frac{t(\tilde{t})}{l} \cosh \frac{t(\tilde{t})}{l} \left(\cosh^2 \frac{t(\tilde{t})}{l} + 2 \sinh \frac{t(\tilde{t})}{l} \right)}{1 + \frac{27A^2}{2m^2 M_{\text{eff}}^2 l^2} \sinh \frac{t(\tilde{t})}{l} \cosh^2 \frac{t(\tilde{t})}{l}} \right)^2 \Bigg\} \\
& +4m^2 M_{\text{eff}}^2 + \frac{189A^2}{l^2} \sinh \frac{t(\tilde{t})}{l} \cosh^2 \frac{t(\tilde{t})}{l}. \tag{72}
\end{aligned}$$

One may find ξ as a function of $t = \zeta$ by using the expression of σ in (39). Then in principle t is given as $t = t(\xi)$. Substituting $t = t(\xi)$ into the expression $\tilde{U}(t)$ in (72), we can find the expression of \tilde{U} as a function of ξ , $\tilde{U} = \tilde{U}(t(\xi))$, which shows, by using the \tilde{U} in (39), the expression of $U(\xi)$. On the other hand, by comparing the expressions of \tilde{V} in (39) and (70), we find $V(\varphi)$. Then by following the procedure from (17) to (19), we get the expression of $F^{(g)}(R^{(g)}, R^{(f)}, e_n(\sqrt{g^{-1}f}))$ and $F^{(f)}(R^{(g)}, R^{(f)}, e_n(\sqrt{g^{-1}f}))$. Thus, the Λ CDM universe can be realized without dark matter. This may suggest that the massive spin two particle might be a dark matter. In the same way, the reconstruction of $F(R)$ bigravity realizing the given cosmological evolution may be done.

V. SUMMARY

In summary, we proposed a bigravity analogue of the $F(R)$ gravity. Our formulation is based on recent ghost-free bigravity theory. The scalar fields are added in both metrics sectors of theory so that after corresponding conformal transformation the scalars become auxiliary ones. Integrating out auxiliary scalars, ghost-free $F(R)$ bigravity follows. It turns out, however, that construction in terms of auxiliary scalars (i.e. when $F(R)$ is given implicitly) is easier to work with. Cosmological equations of the theory under investigation are shown to be consistent. The cosmological reconstruction scheme is developed in detail. It is demonstrated that almost any evolution of physical universe may be realized while second metric solution which often could be flat space exists. The examples of cosmic acceleration which describe phantom, quintessence or Λ CDM universe are presented. The fact that Λ CDM universe may be realized without CDM indicates that massive graviton may play the role of dark matter.

Of course, physical properties of $F(R)$ theory under investigation as well as its other formulations should be further investigated. In this respect, note that it is difficult to get the explicit presentation of usual $F(R)$ gravity which realizes arbitrary cosmological expansion since the reconstruction is made via the solution of the differential equation [20]. In case of $F(R)$ bigravity, we can construct models directly in terms of the auxiliary scalar fields although it is more complicated to give an explicit form of $F(R)$.

We have not discussed the local tests of theory as well as the possibility to generate the fifth force which might not be neglected by experiments. We may construct a model which avoids such problems by using the Chameleon mechanism [21] as in usual $F(R)$ gravity [22]. An analysis by using the post-Newtonian parameter γ was done in [23]. Such an analysis could be also applied to the models proposed in this paper. Moreover, the Vainshtein mechanism [5] might work to suppress the fifth force in general bigravity models. Furthermore, in case of the standard $F(R)$ gravity it was proposed and studied Palatini formulation (Refs. [24–26] and references therein). Such formulation uses different variables set (connections) if compare with metric formulation. Formally, it may lead to the results which are not equivalent with the ones in metric approach. The investigation of massive bimetric $F(R)$ gravity in terms of Palatini-like formulation looks an extremely interesting problem. For instance, does the ghost-free structure of theory survives in Palatini approach? This will be discussed elsewhere.

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